

CPS331 - ARTIFICIAL INTELLIGENCE

Rules of Equivalence for Well-Formed Formulas (WFFs)

We say that two WFFs are equivalent if the first is true when, and only when, the second is true. We denote this by $W1 \leftrightarrow W2$.

For all well-formed formulas $X, X1, X2$, and $X3$:

1. Commutativity: $X1 \wedge X2 \leftrightarrow X2 \wedge X1$
 $X1 \vee X2 \leftrightarrow X2 \vee X1$
2. Associativity: $(X1 \wedge X2) \wedge X3 \leftrightarrow X1 \wedge (X2 \wedge X3)$
 $(X1 \vee X2) \vee X3 \leftrightarrow X1 \vee (X2 \vee X3)$
3. Distributivity: $X1 \wedge (X2 \vee X3) \leftrightarrow (X1 \wedge X2) \vee (X1 \wedge X3)$
 $X1 \vee (X2 \wedge X3) \leftrightarrow (X1 \vee X2) \wedge (X1 \vee X3)$
4. DeMorgan's theorems: $\neg (X1 \wedge X2) \leftrightarrow \neg X1 \vee \neg X2$
 $\neg (X1 \vee X2) \leftrightarrow \neg X1 \wedge \neg X2$
5. Properties of negation: $\neg (\neg X) \leftrightarrow X$
 $X \wedge \neg X \leftrightarrow \text{false}$ (Law of non-contradiction)
 $X \vee \neg X \leftrightarrow \text{true}$ (Law of the excluded middle)
6. Properties of false/true: $X \wedge \text{false} \leftrightarrow \text{false}$
 $X \vee \text{false} \leftrightarrow X$
 $X \wedge \text{true} \leftrightarrow X$
 $X \vee \text{true} \leftrightarrow \text{true}$
7. Definition of implies: $(X1 \rightarrow X2) \leftrightarrow (\neg X1 \vee X2)$
8. Contrapositive law: $(X1 \rightarrow X2) \leftrightarrow (\neg X2 \rightarrow \neg X1)$
9. Properties of quantifiers: $(\exists X) p(X) \leftrightarrow (\exists Y) p(Y)$
 $(\forall X) p(X) \leftrightarrow (\forall Y) p(Y)$

{ i.e. the names of bound variables are not significant }

 $\neg [(\exists X) p(X)] \leftrightarrow (\forall X) \neg p(X)$
 $\neg [(\forall X) p(X)] \leftrightarrow (\exists X) \neg p(X)$

 $(\exists X)[p(X) \vee Q(X)] \leftrightarrow [(\exists X)p(X)] \vee [(\exists Y)q(Y)]$
 $(\forall X)[p(X) \wedge Q(X)] \leftrightarrow [(\forall X)p(X)] \wedge [(\forall Y)q(Y)]$